#1

$$f = \int_0^\infty 4\pi r^2 \rho(r) \frac{\sin(4\pi \sin\theta/\lambda \cdot r)}{4\pi \sin\theta/\lambda \cdot r}\, dr \quad \leftarrow k = 4\pi \sin\theta/\lambda$$

$$= \int_0^\infty 4\pi r^2 \rho(r) \frac{\sin kr}{kr}\, dr \leftarrow \underbrace{I_1 = \int_0^\infty r e^{-\frac{2r}{a}} \sin kr\, dr}_{\text{involved}}$$

To obtain I_1

$$\boxed{\begin{aligned} &(uvw)' = u'vw + uv'w + uvw' \\ &\int_0^\infty u'vw\, dr = [uvw]_0^\infty - \int_0^\infty uv'w\, dr - \int_0^\infty uvw'\, dr \end{aligned}}$$

use this

$$I_1 = \cancel{\left[-\frac{a}{2} e^{-\frac{2r}{a}} r \sin kr\right]_0^\infty} + \frac{a}{2} \underbrace{\int_0^\infty e^{-\frac{2r}{a}} \sin kr\, dr}_{I_3}$$

$$+ \frac{ak}{2} \underbrace{\int_0^\infty e^{-2r/a} r \cos kr\, dr}_{I_2}$$

likewise

$$I_2 = \cancel{\left[-\frac{a}{2} e^{-\frac{2r}{a}} r \cos kr\right]_0^\infty} + \frac{a}{2} \underbrace{\int_0^\infty e^{-\frac{2r}{a}} \cos kr\, dr}_{I_4}$$

$$- \frac{ak}{2} \underbrace{\int e^{-2r/a} r \sin kr\, dr}_{I_1}$$

$$\therefore\ I_1 = \frac{a}{2} I_3 + \frac{ak}{2} I_2, \quad I_2 = \frac{a}{2} I_4 - \frac{ak}{2} I_1$$

keep on doing in the same way

$$I_3 = \cancel{\left[-\frac{a}{2} e^{-\frac{2r}{a}} \sin kr\right]_0^\infty} + \frac{ak}{2} \int_0^\infty e^{-\frac{2r}{a}} \cos kr\, dr$$

$$= \frac{ak}{2} \underbrace{\int_0^\infty e^{-\frac{2r}{a}} \cos kr\, dr}_{I_4}$$

$$I_4 = \left[-\frac{a}{2} e^{-\frac{2r}{a}} \cos kr\right]_0^\infty - \frac{ak}{2} \underbrace{\int_0^\infty e^{-\frac{2r}{a}} \sin kr\, dr}_{I_3}$$

$$= \frac{a}{2} - \frac{ak}{2} I_3$$

$$\therefore\ I_3 = \frac{ak}{2}\left(\frac{a}{2} - \frac{ak}{2} I_3\right) \Rightarrow I_3 = \frac{a^2k/4}{1 + a^2k^2/4}$$

#2

$$\therefore\ I_1 = \frac{a}{2} I_3 + \frac{ak}{2} I_2$$

$$= \frac{a}{2} I_3 + \frac{ak}{2}\left(\frac{a}{2} I_4 - \frac{ak}{2} I_1\right)$$

$$= \frac{a}{2} I_3 + \frac{ak}{2}\left\{\frac{a}{2}\left(\frac{a}{2} - \frac{ak}{2} I_3\right) - \frac{ak}{2} I_1\right\}$$

$$I_1\left(1 + \frac{a^2k^2}{4}\right) = \frac{a^3k}{8} + I_3\,\frac{4a - a^3k^2}{8}$$

$$= \frac{a^3k}{8} + \frac{a^2k/4}{1 + \frac{a^2k^2}{4}} \times \frac{4a - a^3k^2}{8}$$

$$= \frac{a^3 k}{4\left(1 + \frac{a^2k^2}{4}\right)}$$

$$\therefore\ I_1 = \frac{a^3 k}{4\left(1 + \frac{a^2k^2}{4}\right)^2}$$, supposed to give f

$$\times 4\pi \times \frac{2}{\sqrt{4\pi}}\frac{2}{\sqrt{4\pi}} \times \frac{1}{k} \times \frac{1}{a^3}$$ //